Escondido Union High School District

Math 3 (Integrated) with Statistics

EUHSD Board Approval Date: 4/19/16

Honors Extensions Added Spring 2018 for Board Approval: 6/19/18
The EUHSD Mathematics curriculum document outlines specific course expectations in a comprehensive integrated math curriculum. The curriculum documents are designed to be updated annually based on student academic achievement data, research and best practices, and input from stakeholders. The EUHSD mathematics curriculum document contains the following:

A. Course Description
B. Course Guidelines - graduation credit information, transcript information, adopted materials, adopted technology, assessment outline
C. Instructional Materials References
D. Scope and Sequence Map with Essential Standards, Unit Topics, and Key Unit Objectives delineated
E. References to key essential design and implementation documents (see items 1-8 below)

The EUHSD Mathematics program is designed so that all students have access to the rigorous curriculum necessary to graduate high school college and career ready. Pathway options and courses of study will provide students with a rich array of courses designed to meet the needs of all learners from acceleration to intervention. Regardless of a student's entry point, the pathway provides all students access to the highest level of courses offered. The contextualized learning inherent in an integrated program provides students with deeper understanding and improved retention of math concepts. Student-Centered learning provides opportunity for collaboration, communication, and a robust learning environment and provides opportunities for all students to meet the goals of the district’s Instructional Focus at the time of this writing: “All students communicate their thinking, ideas and understanding by effectively using oral, written and/or non-verbal expression.”

A key design consideration in the transition to the new California State Standards is a focus on changes to pedagogy. The instructional shifts guide classroom teaching and learning and the foundation of curriculum and instructional design.

### Instructional Shifts in Mathematics

<table>
<thead>
<tr>
<th>Focus:</th>
<th>Focus requires that we significantly narrow and deepen the scope of content in each grade so that students experience concepts at a deeper level.</th>
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<tbody>
<tr>
<td>Focus strongly where the Standards focus</td>
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</table>
- Instruction engages students through cross-curricular concepts and application. Each unit focuses on implementation of the Math Practices in conjunction with math content.  
- Effective instruction is framed by performance tasks that engage students and promote questions in order to provide a clear and explicit purpose for instruction. |

<table>
<thead>
<tr>
<th>Coherence:</th>
<th>Coherence in our instruction supports students to make connections within and across grade levels.</th>
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<tbody>
<tr>
<td>Think across grades; and link to major topics within grades</td>
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- Problems and activities connect clusters and domains through the art of questioning.  
- A purposeful sequence of lessons build meaning by moving from concrete to abstract, with new learning built upon prior knowledge and connections made to previous learning.  
- Coherence promotes mathematical sense making. It is critical to think across grades and examine the progressions in the standards to ensure the development of major topics over time. The emphasis on problem solving, reasoning and proof, communication, representation, and connections require students to build comprehension of mathematical concepts, procedural fluency, and productive dispositions. |

<table>
<thead>
<tr>
<th>Rigor:</th>
<th>Rigor helps students to read various depths of knowledge by balancing conceptual understanding, procedural skills and fluency, and real-world applications with equal intensity.</th>
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<tr>
<td>In major topics, pursue conceptual</td>
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- Conceptual understanding underpins fluency; fluency is practiced in contextual applications; and applications build conceptual understanding. |
<table>
<thead>
<tr>
<th>understanding, procedural skills and fluency, and application</th>
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<tbody>
<tr>
<td>● These elements may be explicitly addressed separately or at other times combined. Students demonstrate deep conceptual understanding of core math concepts by applying them in new situations, as well as writing and speaking about their understanding. Students will make meaning of content outside of math by applying math concepts to real-world situations.</td>
</tr>
<tr>
<td>● Each unit contains a balance of challenging, multiple-step problems to teach new mathematics, and exercises to practice mathematical skills.</td>
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The EUHSD mathematics curriculum document and all supporting documentation are aligned to the [California State Standards for Mathematics](http://example.com), the Eight Standards for Mathematical Practice, as well as the new CA ELD Standards. These standards will be integrated and delineated within each Unit Plan. A detailed list of resources around which the EUHSD Mathematics curriculum is designed and implemented are as follows:

1. [California State Standards for Mathematics](http://example.com)
2. [Guide to the CASS-M Conceptual Category Abbreviations](http://example.com)
3. [Eight Standards for Mathematical Practice](http://example.com)
4. [California Frameworks for Mathematics](http://example.com)
5. [Smarter Balanced Assessment System Mathematics Assessment Blueprint](http://example.com)
6. [California English Language Development Standards](http://example.com)
7. [University of California Mathematics Pathway FAQ](http://example.com)
8. [Core Plus Instructional Materials](http://example.com)

### California State Standards for Mathematics - Content

The California Standards for high school mathematics (CASS-M) are divided into six conceptual categories, which portray a coherent view of higher mathematics, which cross a number of traditional course boundaries. Each conceptual category is further broken down into domains or clusters of standards that address big ideas - [Guide to the CASS-M Conceptual Category Abbreviations](http://example.com). The conceptual categories are Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. Modeling is best interpreted not as a collection of topics but in relation to other standards, thus Modeling is considered both a conceptual category and a Standard for Mathematical Practice and modeling standards are called within the other conceptual categories, indicated by a (*) symbol.

### Common Core Standards for Mathematical Practice

The CASS-M call for mathematical practices and content to be connected as students engage in mathematical tasks. These connections are essential to support the development of students’ broader mathematical understanding - students who lack understanding of a topic may rely too heavily on procedures. The 8 Math Practice standards must be taught as carefully and practiced as intentionally as the Standards for Math Content. Neither should be isolated from the other; effective mathematics instruction occurs when the two halves of the CASS-M come together as a powerful whole (CASS-M, 2013).
Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**

   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

   Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical

4. **Model with mathematics.**

   Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical
situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Math 3 with Statistics - Course Description

Math 3 with Statistics is the third of an integrated and investigative mathematics program designed to use patterns, modeling, and conjectures to build student understanding and competency in mathematics. The Math 3 with Statistics course is designed to formalize and extend student understanding of Algebra and Functions, Geometry and Trigonometry, Statistics and Probability, and Discrete Mathematics. The critical areas of focus include: mathematical reasoning in various contexts; linear programming as a tool for problem-solving; extending understanding of congruence and similarity of geometric shapes; investigating and understanding variability in data sets; extending understanding of functions to include polynomial, rational, and inverse functions; symmetry of circles and circular motion; and developing student understanding of sequential change more fully. The Math 3 with Statistics curriculum addresses each of the conceptual categories outlined by the California State Standards for Mathematics: number and quantity; algebra, functions, geometry, statistics and probability, and modeling. The curriculum includes attention to each of the Eight Standards for Mathematical Practice. Instructional materials and classroom experiences provide students with a rich array of resources and technology designed to support student mastery of specific concepts and both procedural and conceptual understanding while building essential 21st Century skills designed for college and career readiness. Assessments are outlined in the Scope and Sequence document. Each unit will culminate in a district-wide common assessment (year 1 implementation will use Core-Plus provided assessments).

Honors Option for Math 3 with Statistics

The primary goals of the EUHSD Mathematics program are as follows: first, ensure students develop a deeper understanding of mathematics; second, prepare students to take on more advanced studies in math; third, personalize the pursuit of honors and advanced coursework; fourth, to focus on the level of rigor evidenced by the work produced by students.

Accordingly, students who elect to participate in and produce honors level work will receive honors credit. Students who earn the honors designation in these courses will produce work that addresses more sophisticated questions, delve more deeply into the mathematics of the course, and satisfy the California State Standards with added rigor. The honors modification applied to some course assignments will broaden the scope or deepen the examination of the course content.

Within the first 20 days of the semester, students who elect to challenge themselves with honors work will complete an Honors Option Registration Contract, signed by Student, Teacher, and Parent/Guardian (see attached). Students who opt-in and later elect not to pursue the honors option for the course must complete a Petition for Release from Honors Option Contract by the 8th week of the semester, to be signed by Student, Teacher and Parent/Guardian (also attached).

Students electing the Honors Option will have differentiated homework, required projects, portfolios due each semester, required participation in a demonstration of learning once per semester, and will be required to demonstrate understanding on assessment extension problems.
# Math 3 with Statistics Course Requirements

<table>
<thead>
<tr>
<th>Course Length: Year-Long</th>
<th>Grade Level: Grade 9-12</th>
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<tbody>
<tr>
<td>UC/CSU Requirement: Approved by UC/CSU as a “c” mathematics course</td>
<td>Graduation Requirement: Students will receive (5) math credits per successfully completed semester for a total of (10) math credits for the year.</td>
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<tr>
<th>Course Number Semester A:</th>
<th>Transcript Name Semester A:</th>
<th>Course Number Semester B:</th>
<th>Transcript Name Semester B:</th>
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<tbody>
<tr>
<td>(P) 2261</td>
<td>(P): MATH 3 W/STATS A (P)</td>
<td>(P) 2262</td>
<td>(P): MATH 3 W/STATS B (P)</td>
</tr>
<tr>
<td>(H) 2285</td>
<td>(H): MATH 3 W/STATS A (H)</td>
<td>(H) 2286</td>
<td>(H): MATH 3 W/STATS B (H)</td>
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<tr>
<td>(SE) 2263</td>
<td>(SE): MATH 3 W/STATS A (P) SE</td>
<td>(SE) 2264</td>
<td>(SE): MATH 3 W/STATS B (P) SE</td>
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Honors course will be the “Opt-in” model, taught within the same classroom. We will not be registering any students for these courses until after the beginning of the school year. Students will be required to sign a contract, as will their parents.

Number of Credits Semester A: five (5) *Honors Not Weighted Number of Credits Semester B: five (5) *Honors Not Weighted

Required Prerequisites: N/A

Recommended Prerequisites: Math 1 and Math 2 or Algebra 1 and Geometry

Board Approval Date/Curriculum: 4/19/16

Board Approval Date/Textbooks: 4/19/16

Supplemental Resource/s:
- All adopted ancillary materials within the Core-Plus instructional program

Supplemental Technology Resource/s:
- Graphing Calculator
- *CPMP Tools (online resource)*

Assessment/s:
- Year 3 - All EUHSD Math 3 teachers will give common assessments, which accompany the Core-Plus Mathematics Program (lesson quizzes, unit tests, common district final exams).
## Unit 1 - Reasoning and Proof: Math 3 Scope and Sequence

### Unit 1 Description
- Students begin to develop an understanding of mathematical reasoning in geometric, algebraic, and statistical contexts.

### Unit 1 Learning Goals:
- Recognize the differences between, as well as the complementary nature of, inductive and deductive reasoning.
- Develop some facility in analyzing and producing deductive arguments in everyday contexts and in geometric, algebraic, and statistical contexts.
- Know and be able to use the relations among the angles formed when two lines intersect, including the special case of perpendicular lines.
- Know and be able to use the necessary and sufficient conditions for two lines to be parallel.
- Use symbolic notation to represent numerical patterns and relationships and use rules for transforming algebraic expressions and equations to prove those facts.
- Distinguish between sample surveys, experiments, and observational studies; know the characteristics of a well-designed experiment.
- Use statistical reasoning to decide whether one treatment causes a better result than a second treatment.

### Unit 1 Assessment:
- **Assessment for Learning** - Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm-ups, journaling, questioning, and self-grading.

### Unit 1 Focus Standards:

#### Algebra
- **A-SSE.1** - Interpret the structure of expressions that represent a quantity in terms of its context, such as factors, terms, and coefficients and interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-REI.1** - Understand solving equations as a process of reasoning and explain the reasoning. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### Geometry
- **G-CO.1, 2, 4, 6, 9, 12** - Know precise definitions of angle, circle, perpendicular line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc; Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Prove theorems about lines and angles.

### Learning Objectives
- Students will...
  - Recognize the role of inductive reasoning in making conjectures and recognize the limitations of inductive reasoning.
  - Recognize the need for proof and be able to create a simple deductive argument to prove a mathematical assertion.
  - Create a counterexample to prove a claim is false.
  - Write if-then statements and their converses and use if-then reasoning patterns in arguments.
  - Use inductive reasoning to develop line reflection assumptions and use deductive reasoning to justify line reflection properties.
  - Know and be able to use the angle relationship theorems involving two intersecting lines.
  - Know and be able to use the theorems justifying the construction of a line perpendicular to a given line through a given point and the construction of a line parallel to a given line through a given point.
  - Know and be able to use the angle relationship theorems involving two parallel lines cut by a transversal.

### Sample Investigations

#### Reasoned Arguments-
In this investigation, students learn about the role of evidence and assumptions in making and proving claims.

#### Intersecting Lines and Angles-
In this investigation, students will begin to develop the skills needed to create sound deductive arguments in the context of exploring and trying to understand relationships between angles created when two lines intersect.

#### Algebraic Expressions-
In this investigation, students examine the logic behind the number magic trick and asking students to analyze and develop some similar ideas through algebraic reasoning.

#### Design of Experiments-
In this investigation, students learn the vocabulary of the design of experiments and to learn that...
### Unit 1

- **Description**
- **Learning Goals**
- **Key Assessments**

#### Observations
Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are internalizing the content and procedures.

#### Assessment of Learning
- Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

#### Writing to Learn
Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

#### Performance Assessments
There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

#### Common Assessments
- Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

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**Statistics and Probability:**

**S-ID.1*, 2*, 3*:** Represent data with plots on the real number line. Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

**S-IC.3*, 5*:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

**Modeling**

*all standards marked with asterisks are modeling standards

**Unit 1 Related Standards:**

**A-CED.4*:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**A-REI.10:** Understand the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**A-SSE.1*, 3*, 4*:** Interpret expressions that represent a quantity in terms of its context. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Derive the formula for the sum of a finite geometric series and use the formula to solve problems.

**F-LE.1*, 4*:** Distinguish between situations that can be modeled with linear functions and with exponential functions. Use the unit circle to explain symmetry by a transversal and their converses.

**Learning Objectives**

- Know and be able to use the angle sum theorem and the exterior angle theorem for triangles.
- Use algebraic notation - letters, expressions, equations, and inequalities to represent general patterns and relationships among variables.
- Use algebraic transformations of expressions, equations, and inequalities to establish general propositions about quantitative relationships.
- Know the characteristics of a well-designed experiment.
- Understand the placebo effect.
- Under the hypothesis of no treatment effect, construct an approximate sampling distribution for the difference of two means by re-randomizing.
- Use a randomization test to decide if an experiment provides statistically significant evidence that one treatment is more effective than another.

**By Chance or from Cause**

In this investigation, students will learn how to perform a randomization test to decide whether one treatment is more effective than another.

**Honors Extensions**

- **Lesson 1:** OYO #E28, p. 26
- **Lesson 2:** OYO #E23, E24, or E25, p. 47-48
- **Lesson 3:** OYO #E30 or E32, p. 71-72
- **Lesson 4:** OYO #E15, E16, or E17, p. 98-99

**Project:** Take Home Test
(odd and even) and periodicity of trigonometric functions.

**F-BF.2*:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

**F-TF.8:** Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to find sin, cos, tan, given sin, cos, or tan and the quadrant of the angle.

**G-CO.10:** Prove theorems about triangles.

**G-MG.1*:** Use geometric shapes, their measures, and their properties to describe objects.

**G-SRT.10:** Prove the Laws of Sines and Cosines and use them to solve problems.

**S-IC.4*, 6*:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Evaluate reports based on data.
Math 3 with Statistics Scope and Sequence
Unit 2 - Inequalities and Linear Programming

Unit 2 Description
- This unit focuses on the formulation and solution of inequalities in one variable with special emphasis on quadratic inequalities and students are introduced to the linear programming technique for solving such inequalities.

Unit 2 Learning Goals:
- Write inequalities to express questions about functions of one or two variables.
- Solve quadratic inequalities in one variable, and give examples showing how extraneous solutions may arise.
- Solve quadratic equations in one variable by completing the square, by inspection, taking square roots, the quadratic formula and factoring; recognize when the quadratic formula gives complex solutions and write them as a +/- bi for real numbers a and b.
- Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Explain why the x-coordinates of the points where the graphs of the equations intersect are the solutions of the corresponding equation in one variable.
- The problems of the investigation focus on the solution of quadratic inequalities by algebraic reasoning with the conceptual support of graphic images.

Unit 2 Assessments:
- Assessment for Learning- Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are internalizing the content and procedures.
- Assessment of Learning- Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make connections and apply concepts in new situations.

Unit 2 Focus Standards:
- Algebra
  A-CED.1*, 2*, 3*: Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
  A-REL2, 4, 6, 7, 10, 11*, 12: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Solve quadratic equations in one variable by completing the square, by inspection, taking square roots, the quadratic formula and factoring; recognize when the quadratic formula gives complex solutions and write them as a +/- bi for real numbers a and b. Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Explain why the x-coordinates of the points where the graphs of the equations intersect are the solutions of the corresponding equation in one variable.

Students will...
- Write inequalities to express questions about functions of one or two variables.
- Given a graph of one or more functions, solve inequalities related to the function (s).
- Solve quadratic inequalities in one variable by solving the corresponding equation algebraically and reasoning about the graph of the related function (s).
- Describe the solution set of an inequality in one variable symbolically, as a graph on a number line, and using interval notation.
- Graph the solution set of a linear inequality in two variables.
- Graph the solution set of a system of inequalities in two variables.
- Solve linear programming problems involving two independent variables.

Getting the Picture-
This investigation lays a conceptual foundation for graphic thinking about inequalities that will be a useful complement to algebraic methods that are developed in the subsequent investigations.

Quadratic Inequalities-
The problems of the investigation focus on the solution of quadratic inequalities by algebraic reasoning with the conceptual support of graphic images.

Linear Programming - A Graphic Approach
This investigation leads students to optimal solutions of linear programming problems using only graphs of the constraints and informal exploration of values for the objective function.

Linear Programming - Algebraic Methods-
In this investigation, students express the constraints for the linear programming problems from the previous investigation with inequalities and use these symbolic expressions to create...
sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

Writing to Learn- Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

Performance Assessments- There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

Common Assessments Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

- **Y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, include cases where the functions are linear, polynomial, rational, absolute value, exponential and logarithmic. A-SSE.1*, 3*: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients; interpret complicated expressions by viewing one or more of their parts as a single entity. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression - factor to reveal zeroes, complete the square to reveal max or min values, use properties of exponents to transform expressions for exponential functions.**

- **Functions**
  - F-IF.1, 2, 4*, 5*, 7*: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include: intercepts; intervals where the function is**
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increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-LE.5*: Interpret the parameters in a linear or exponential function in terms of a context.

Modeling

*all standards marked with asterisks are modeling standards

### Unit 2 Related Standards:

F-TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-LE.1*: Distinguish between situations that can be modeled with linear functions and with exponential functions.

G-SRT.11: (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
## Math 3 with Statistics Scope and Sequence
### Unit 3 Similarity and Congruence

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<tr>
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<th>Unit 3 Focus Standards:</th>
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</table>
| - This unit extends student understanding and skill in the use of similarity and congruence relations to solve problems involving shape and size. | **Geometry**  
G-C.1: Prove that all circles are similar.  
G-C.2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Explain how the criteria for similarity of figures. | **Students will…**  
- Identify similar polygons and determine the scale factor of similar polygons.  
- Review and extend understanding of the Laws of Sines and Cosines.  
- Know and be able to use the three theorems providing sufficient conditions to prove triangles are similar (SSS, SAS, AA).  
- Continue to develop the ability to write both synthetic and coordinate arguments.  
- Discover and prove properties of size transformations (dilations) using a synthetic approach.  
- Understand congruence of figures as a special case of similarity of figures.  
- Know and be able to use the four theorems providing sufficient conditions to prove triangles are congruent (SSS, SAS, AAS, ASA).  
- Know and be able to use properties of the incenter, circumcenter, and centroid of a triangle.  
- Continue to develop the ability to write both synthetic and coordinate arguments.  
- Know and be able to use both necessary and sufficient conditions for congruence in terms of rigid motions to decide if they are congruent. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Explain how the criteria for | **When Are Two Polygons Similar?**  
This investigation develops the concept of similar polygons. The focus is on testing for similarity of triangles.  
**Reasoning with Similarity Conditions**  
In this investigation, students consider how similarity and proportionality are used in a variety of applied situations and in proving mathematical relationships such as the Midpoint Connector Theorem.  
**Congruence in Triangles**  
In this investigation, students will explore three centers of triangles and their properties - the circumcenter (The center of a circle circumscribed about the triangle), the incenter (the center of a circle inscribed in the triangle), and the centroid (the center of gravity).  
**Congruence and Similarity**  
**A Transformation Approach**  
In this investigation, students will re-examine reflections, translations, and rotations from a synthetic perspective. |

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<th>Unit 3 Learning Goals:</th>
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| - Build skill in using inductive and deductive reasoning to first discover and then prove geometric relationships and properties based on similarity and congruence of triangles.  
- Develop facility in producing deductive arguments in geometric situations using synthetic, coordinate, and transformational methods.  
- Know and be able to use triangle similarity and congruence theorems.  
- Know and be able to use properties of special centers of triangles.  
- Know and be able to use the necessary and sufficient conditions for quadrilaterals to be (special) parallelograms and for special quadrilaterals to be congruent.  
- Know and be able to use properties of size transformations and rigid transformations (line reflections, translations, and rotations) to prove sufficient conditions for congruence of triangles and solve problems. | | |

| Unit 3 Assessments: | | |
|-----------------------| | |
| - **Assessment for Learning**- Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are | | |

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**Math 3 with Statistics Scope and Sequence**  
**Unit 3 Similarity and Congruence**  
This unit extends student understanding and skill in the use of similarity and congruence relations to solve problems involving shape and size. Students will learn to build skill in using inductive and deductive reasoning to first discover and then prove geometric relationships and properties based on similarity and congruence of triangles. They will develop facility in producing deductive arguments in geometric situations using synthetic, coordinate, and transformational methods. Students will know and be able to use triangle similarity and congruence theorems. They will know and be able to use properties of special centers of triangles. Students will know and be able to use the necessary and sufficient conditions for quadrilaterals to be (special) parallelograms and for special quadrilaterals to be congruent. They will know and be able to use properties of size transformations and rigid transformations (line reflections, translations, and rotations) to prove sufficient conditions for congruence of triangles and solve problems. Assessment for Learning - Informal grading will help students to improve their quality of work. Assessment for Learning - Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are.
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- **Assessment of Learning**: Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

- **Writing to Learn**: Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

- **Performance Assessments**: There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

- **Common Assessments**: Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

- **Internalizing the content and procedures**.

- **Assessment of Learning**: Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

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- **Conditions for quadrilaterals to be (special) parallelograms**.
  - Know and be able to use the Midpoint Connector Theorems for Triangles and Quadrilaterals.
  - Discover and prove properties of rigid transformations using a synthetic approach.
  - Know and use the key ideas of rigid transformations in providing sufficient conditions for congruence of triangles and composites of a size transformation and rigid transformations in providing sufficient conditions for similarity of triangles.

- **Honors Extensions**
  - Lesson 1: OYO #24, p. 190
  - Lesson 2: OYO #E28, p. 223

- **Project**: Miniature Golf Mania
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● Related Standards | ● Learning Objectives | ● Sample Investigations and Assignments  
● Honors Extensions |

**Modeling**  
*all standards marked with asterisks are modeling standards*

**Unit 3 Related Standards:**  
A-SSE.2: Use the structure of an expression to identify ways to rewrite it.  
G-SRT.8*, 10, 11: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Derive and use the trigonometric ratios for special right triangles (30°, 60°, 90°) and 45°, 45°, 90°). Prove the Laws of Sines and Cosines and use them to solve problems. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  
G-GPE.4: Use coordinates to prove simple geometric theorems algebraically.
# Math 3 with Statistics Scope and Sequence

## Unit 4 Samples and Variation

### Unit 4 Samples and Variation:
- In this unit, students will investigate how understanding variability in outcomes helps both pollsters, manufacturers improve their products, and the statistical methods used to do so.

### Unit 4 Learning Goals:
- Understand the standard deviation as a measure of variability and the normal distribution as a model of variability.
- When appropriate, fit a normal distribution to a set of data and use it to estimate percentages.
- Construct binomial probability distributions, find, and interpret expected values.
- Use a random sample to make an inference about whether a specified proportion \( p \) is plausible as the population parameter.
- Use concepts of probability to create strategies for product testing.
- Understand the Central Limit Theorem and how it is applied to product testing.

### Unit 4 Assessments:
- **Assessment for Learning:** Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are internalizing the content and procedures.

### Unit 4 Focus Standards:
- **Statistics and Probability:**
  - [S-ID.1*: 2*: 3*: 4*]: Represent data with plots on the real number line (dot plots, histograms, and box plots). Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
  - [S-IC.1*]: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
  - [S-MD.2*, 3*, 4*, 7*]: (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. (+) Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).

### Students will…
- Describe characteristics of a normal distribution.
- Understand that the number of standard deviations from the mean is a measure of location.
- When appropriate, fit a normal distribution to a set of data and use it to estimate percentages.
- Use simulation to construct approximate binomial probability distributions.
- Predict the shape of a binomial distribution.
- Find and interpret expected values and standard deviations of binomial distributions.
- Use a fitted normal distribution to estimate probabilities of events in binomial situations.
- Use a random sample to make an inference about whether a specified proportion \( p \) is plausible as the population parameter.
- Recognize when the mean and standard deviation change on a plot-over-time.
- Use concepts of probability to create strategies for product testing.
- Compute the probability of a false alarm on a set of readings, that is, the probability that a test will give an out-of-control signal for a process that is under control.

### Standardized Values:*
- In this investigation, students will learn how to use the standard deviation as the unit of measurement in measuring how far a value is from the mean of a normal distribution.

### Shapes, Center, and Spread:*
- In this investigation, students will learn how to construct a binomial distribution for a given probability of success \( p \) and number of trials \( n \) and to estimate the distribution’s shape, mean, and standard deviation.

### Binomial Distributions and Making Decisions:*
- In this investigation, students learn to recognize which events are likely and which are unlikely (rare events) to happen in a binomial situation. They learn to use the results from a random sample to decide whether it is plausible that a specified proportion is the correct proportion of successes for the binomial population from which the sample was taken.

### Out of Control Signals:*
- In this investigation, students apply their knowledge of probability and the normal distribution to study...
**Assessment of Learning** - Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

**Writing to Learn** - Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

**Performance Assessments** - There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

**Common Assessments** - Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

**Focus Standards**

| N-Q.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| F-IF.7* | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| S-CP.1*, 2*, 3*: | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. |
| S-ID.8*: | Compute (using technology) and interpret the correlation coefficient of a linear fit. |
| the tests commonly used by industry to signal that a manufacturing process has gone out of control. |

**False Alarms** - In this investigation, students compute the probability that, when monitoring a process in control, an out-of-control signal will be given on the next set of observations by a particular test. Such an out-of-control signal for a process that is in control is called a false alarm.

**Honors Extensions** -

| Lesson 1: | OYO #E20, p. 255 |
| Lesson 2: | OYO #E19, p. 280 |
| Lesson 3: | skip |

**Project:** Analyzing Data or Local Warming (student choice)
# Math 3 with Statistics Scope and Sequence
## Unit 5 - Polynomial and Rational Functions

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<tr>
<td><strong>Unit 5 Polynomial and Rational Functions:</strong></td>
<td>● This unit extends students’ repertoire of useful function models to the family of polynomials and to analyze the new possibilities for table and graph patterns, equations, inequalities, and algebraic operations that arise from consideration of polynomials of degree greater than two.</td>
<td><strong>Unit 5 Focus Standards:</strong></td>
<td><strong>Number &amp; Quantity:</strong></td>
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<td><strong>Students will…</strong></td>
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<td><strong>Unit 5 Learning Goals:</strong></td>
<td>● Recognize patterns in problem conditions and in data plots that can be described by polynomial and rational functions.</td>
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<td>N-CN.1, 2, 7: Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Solve quadratic equations with real coefficients that have complex solutions.</td>
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<td>● Model problem situations using polynomial functions.</td>
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<td>● Write polynomial and rational functions rules to describe patterns in graphs, numerical data, and problem conditions.</td>
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<td>Algebra</td>
<td>A-SSE.1*, 2, 3*: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. Use the structure of an expression to identify ways to rewrite it. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeroes of the function it defines. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expressions for exponential functions.</td>
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<td>● Identify patterns relating rules and graphs of polynomial functions - connecting polynomial degree to local maximum and local minimum values and zeroes.</td>
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<td>● Use table, graph, or symbolic representations of polynomial and rational functions to answer questions about the situations they represent. Evaluate functions, solve equations and inequalities, identify local min/max points and asymptotes.</td>
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<td>A-CED.2*: Create equations (1 or 2 variables) and inequalities (1 variable only) that describe numbers or relationships, including ones with absolute value (CA), and use them to solve problems. Graph equations on coordinate axes with labels and scales.</td>
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<td></td>
<td>● Add, subtract, and multiply polynomials - connecting polynomial functions - useful function models to the family of polynomials - connecting polynomial functions to degrees of sums, differences, and products.</td>
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<td>● Rewrite polynomial and rational expressions in equivalent forms by expanding or factoring, by combining like terms, and by removing common factors in numerator and denominator of rational expressions.</td>
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<td>A-APR.1, 2, 3, 6, 7: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. Identify zeroes of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial; Rewrite simple rational expressions in different types of graph possibilities.</td>
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<td>● Find zeroes of polynomial functions and create polynomial functions with prescribed zeroes.</td>
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<td></td>
<td>● Add, subtract, and multiply polynomial and rational expressions and functions.</td>
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<td>● Express quadratic function rules in vertex form.</td>
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<td>● Extend understanding and skill in work with quadratic functions to include completing the square, interpreting vertex form, and proving the quadratic formula.</td>
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<td>● Use vertex form of quadratic expressions to solve quadratic equations and locate the vertex of parabolic graphs.</td>
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<td>● Recognize and calculate complex number solutions of quadratic equations.</td>
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<td>● Use completing the square to prove the quadratic formula.</td>
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<td>● Use the quadratic formula to analyze solution possibilities for quadratic equations and indicate the rationale for extending the number system to include complex numbers.</td>
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### Sample Investigations
- [Polynomials-Modelling with Polynomial Functions-](#) In this investigation, students will learn how to model complex graphical patterns with polynomial functions of degree 3 and 4. Then they will explore a variety of polynomials to discover the type of graph possibilities.
- [Zeroes and Products of Polynomials-](#) In this investigation, students will use the factored form of a polynomial function to determine its zeroes and will use factors to construct a polynomial function with prescribed zeroes.
- [Completing the Square-](#) In this investigation, students will be introduced to the vertex form of a quadratic expression $a(x-h)^2 + k$. This will help them to graph quadratic equations without the use of technology.
- [Domains and Graphs of Rational Functions-](#) In this investigation, students will begin work with rational functions. Students will analyze situations using...
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<td><strong>In this investigation, students will continue working with rational expressions, specifically simplifying the expressions to less complicated forms.</strong></td>
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**Functions**

- **F-IF.1, 2, 4*, 5*, 7*, 8**: Understand that a function from one set to another set assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quotients of known functions as models of the variation that is involved.
Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

- **Performance Assessments** - There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

- **Common Assessments** - Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

### Focus Standards

- **N-CN.4, 9**: (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number; (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- **N-VM.8**: (+) Add, subtract, and multiply matrices of appropriate dimensions.
- **A-CED. 1**: Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **F-BF.3**: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs.

### Related Standards

- **N-CN.3**: Know the complex conjugate property, and know and use the relationship between a complex number and its conjugate.
- **N-CN.1**: Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has exactly one complex conjugate.
- **N-CN.7**: Solve quadratic equations with real coefficients that have complex solutions.
- **N-CN.8**: Extend polynomial identities to the complex numbers.
- **N-CN.2**: Use the relation $i^2 = -1$ and the commutativity of multiplication to derive the complex conjugate property: $(a + bi)(a - bi) = a^2 + b^2$ for any real numbers $a$ and $b$.
- **N-CN.9**: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- **N-VM.10**: Understand that the complex numbers form an infinite-dimensional vector space over the reals, and use this to solve problems involving Euclidean geometry.
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● Honors Extensions |

Experiment with cases and illustrate an explanation of the effects on the graph using technology.  
**G-MG.3***: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  
**G-SRT.5, 6**: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  
**S-ID.6***: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
**Math 3 with Statistics Scope and Sequence**

**Unit 6 - Circles and Circular Functions**

- **Unit 6 Description**
- **Unit 6 Learning Goals**
- **Unit 6 Key Assessments**

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**Unit 6 Focus Standards:**

**Functions**

**F-BF.3:** Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**F-TF.1, 2, 3, 5:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Graph all 6 basic trigonometric functions. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosine, and tangent for \( \pi-x, \pi+x \), and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**Geometry**

**G-MG.1**, **3:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder); Apply geometric methods to solve design problems (e.g.,

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**Students will…**

- **Discover and prove that a line tangent to a circle is perpendicular to the radius at the point of tangency and that the two tangent segments to a circle from the same external point are congruent.**
- **Discover, prove, and apply the relationships among the measures of central angles, their chords, and their arcs.**
- **Discover, prove, and apply the properties relating to a radius, a chord, and the midpoint and perpendicular bisector of the chord.**
- **Discover, prove, and apply the Inscribed Angle Theorem and the property that inscribed angles that intercept the same or congruent arcs are congruent.**
- **Analyze situations involving pulleys or sprockets to study circular motion.**
- **Use sine and cosine functions to model aspects of circular motion.**
- **Discover and prove that the sine and cosine functions that the sine and cosine functions from trigonometry can be used to track the position of a point on the circumference of a circular object as that object rotates about a point.**
- **Patterns of Periodic Change:** Students will investigate the more general functions \( y = a \sin bx + c \) in greater detail. They will look at how changes in amplitude and period express themselves in the equations and graphs of sine and cosine curves.
will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

**Writing to Learn**- Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

**Performance Assessments**-There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

**Common Assessments**- Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  
**G-C.2, 3, 4, 5:** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (+) Construct a tangent line from a point outside a given circle to the circle. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.  
**G-CO.1, 12, 13:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  
**Modeling**

*all standards marked with asterisks are modeling standards

of the arc that the angle intercepts.
- Define sine and cosine as functions of real numbers and analyze the resulting periodic graphs.
- Use the sine and cosine functions to model periodic patterns of change in various physical phenomena.

**Honors Extensions** -  
**Lesson 1:** OYO #E30, p. 417  
**Lesson 2:** OYO #E26 or E30, p. 447, student choice  
**Project:** Gears and Circular Motion or Periodic Patterns – student choice
### Unit 6 Related Standards:

- **N-Q.1**: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N-VM.11, 12**: (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors; (+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- **F-IF.7a**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **G-CO.6**: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **G-GPE.4**: Use coordinates to prove simple geometric theorems algebraically.
- **G-SRT.5, 7**: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures; Explain and use the relationship between the sine and cosine of complementary angles.
- **S-MD.7**: (+) Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).
### Unit 7 - Recursion and Iteration

**Unit 7 Description**
- This unit develops student understanding of sequential change more fully. It builds on previous work with recursion and iteration to study such change.

**Unit 7 Learning Goals**
- Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change.
- Formalize and consolidate previous study of NOW-NEXT rules, particularly through the use of subscript notation and the introduction of recursive formulas.
- Understand and apply arithmetic and geometric sequences and series.
- Understand and apply finite differences tables.
- Explore function iteration and, in the process, informally introduce function composition.
- Understand and apply recursive formulas.
- Review linear, exponential, and polynomial models from a recursive perspective.

**Unit 7 Assessments**
- **Assessment for Learning** - Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the pulse of the classroom and see how students are internalizing the content and procedures.
- **Assessment of Learning** - Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The

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#### Sample Investigations

**Modeling Population Change**
- The main focus of this investigation is to examine the behavior of recursive formulas, starting with the familiar NOW-NEXT rules. This is both a practical and puzzling investigation and students may be surprised to discover that the long-term population of the fishpond is not dependent on the starting population but only on the die-off rate and the restocking amount.

**Arithmetic and Geometric Sequences**
- The goal of this investigation is for students to realize that arithmetic sequences are connected to linear functions and that, in those cases, \( a \) can be expressed using both function and recursive formulas.

**Finite Differences**
- In this investigation, students use finite differences tables to help them determine the polynomial function that generates a sequence.

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### Unit 7 Focus Standards:

#### Algebra
- **A-CED.2***: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A-REI.2, 10**: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise; Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **A-SSE.1*, 4**: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

#### Functions
- **F-BF.1*, 2**: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. (+) Compose functions. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

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### Unit 7 Learning Objectives

**Students will...**
- Use iteration and recursion to model real-world situations involving sequential change.
- Understand the basic concepts of recursive formulas, particularly those of the form \( A_n = A_{n+1} + b \).
- Understand the effects of changing certain parameters on the long-term behavior of recursive formulas and the situations they model.
- Use subscript notation and spreadsheet software to represent formulas that use the words NOW and NEXT to take advantage of this notation and spreadsheet software to analyze recursive formulas more efficiently.
- Understand arithmetic sequences and their connections to linear functions, using recursive formulas, function formulas, and applications.
- Understand geometric sequences and their connections to exponential functions, using recursive formulas, functions formulas, and applications.
- Understand and apply arithmetic and geometric series (sums of sequences).
contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

- **Writing to Learn** - Students will keep a mathematics journal to write about their thinking during investigations and projects. Students will be required to justify their reasoning during any investigation and be able to summarize their conclusions. Writing about the mathematics will encourage students to construct viable arguments and critique the reasoning of others.

- **Performance Assessments** - There will be smaller projects and key assignments during the year in which students will show their mastery of the content through a project that allows students to display their learning using multiple representations.

- **Common Assessments** - Each unit will culminate in a district-wide common unit assessment (year 1 implementation will use Core-Plus provided assessments).

### Focus Standards

- **F-IF.1, 2, 3:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \). Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F-LE.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### Modeling

- All standards marked with asterisks are modeling standards

### Unit 7 Related Standards:

- **N-VM.8, 9:** (+) Add, subtract, and multiply matrices of appropriate dimensions; (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **A-CED.1:** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple

### Learning Objectives

- Use finite differences tables to find function formulas for certain recursive formulas and to describe the connection between such tables and polynomial functions.
- Use linear, exponential, and polynomial functions to model discrete situations.
- Iterate functions and describe the resulting patterns, the long-term behavior in particular.
- Describe the connection between function iteration and recursive formulas.
- Analyze long-term behavior when iterating linear functions, using graphical iteration, numerical iteration, and algebraic methods, including fixed-point analysis and connections to slope.

### Sample Investigations and Assignments

- **Play It Again...and Again** - This investigation is designed to establish the connection between evaluating recursive formulas and iterating functions and to give students an experience of the richness of long-term behavior that is possible when you iterate even simple functions.

### Honors Extensions

- **Lesson 1:** OYO #E15 or #E18, p. 475
- **Lessons 2 – 3 are omitted per unit plan.**

### Project:

- Take Home Test - spreadsheets recommended
**rational and exponential functions.**

**A-REI.1, 6, 8, 9:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (+) Represent a system of linear equations as a single matrix equation in a vector variable. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

**S-ID.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Informally assess the fit of a function by plotting and analyzing residuals. Fit a linear function for a scatter plot that suggests a linear association.

**S-MD.1, 2:** (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
## Math 3 with Statistics Scope and Sequence
### Unit 8 - Inverse Functions

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### Unit 8 - Inverse Functions:
- This unit develops student understanding and skill in use of inverse functions, especially the square root, logarithm, and inverse trigonometric functions.

### Unit 8 Learning Goals:
- Discover conditions that guarantee existence of inverse functions.
- Develop strategies for finding rules for inverse functions.
- Use inverse functions to solve problems of coding and decoding information.
- Develop the definition and important properties of inverses for exponential functions.
- Use logarithms to solve exponential equations.
- Develop the definition and important properties of inverse sine, cosine, and tangent functions.
- Use inverse trigonometric functions to solve trigonometric equations and inequalities and the problems in which those equations and inequalities arise.

### Unit 8 Assessments:
- **Assessment for Learning:** Feedback given in many forms such as direct teacher conversations and revision suggestions, mini-quizzes, peer grading rubrics, and self-grading will help students to improve their quality of work and deepen their understanding. Informal daily assessments will be used daily by teachers through warm ups, journaling, questioning, and observations. Informal observations are important for teachers to conduct daily to get the

### Unit 8 Focus Standards:

#### Algebra

- **A-CED.1**: Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

#### Functions

- **F-BF.1**: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. (+) Compose functions. Find inverse functions; Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse. (+) Verify by composition that one function is the inverse of another. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (+) Produce an invertible function from a non-invertible function by restricting the domain. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

- **F-IF.1**, **4**, **5**, **8**: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph

### Students will…
- Solve problems involving direct and inverse variation.
- Discover conditions that guarantee existence of an inverse for a given function.
- Develop and use strategies for recognizing invertible functions from study of tables of values and/or graphs of those functions.
- Develop and use strategies for finding rules of inverses for linear and power functions.
- Express a positive number as a power of 10.
- Define and evaluate common logarithms.
- Use logarithms to solve exponential equations.
- Develop and use basic properties of the logarithmic function.
- Know and be able to use the definition of the inverse sine, inverse cosine, and inverse tangent functions.
- Know and be able to use properties of the inverse sine, inverse cosine, and inverse tangent functions.
- Use the inverse functions, to find one solution (when one exists) of \( a \ast f(bx) + c = d \), where \( f(x) \) is the sine, cosine, or tangent.
- Express the general solutions of a trigonometric equation in forms

### Coding and Decoding Message-
- This investigation highlight the point that symbols in computer codes must have unique numeric codes and show ways of making the coding algorithms difficult to crack so that information is secure as sent.

### Covering All the Bases-
- In this investigation, students will use what they learned about logarithms previously to solve exponential equations in which the natural base is not 10.

### Properties of Logarithms-
- The goal of this investigation is to develop student understanding and skill in use of the basic properties of the logarithmic function.

### The Ups and Downs of Sine-
- The problems of this investigation help students notice that the sine function defined over all real numbers has no inverse.
pulse of the classroom and see how students are internalizing the content and procedures.

- **Assessment of Learning** - Small quizzes, larger unit tests, and benchmark exams will assess what students have learned. Small quizzes and unit tests will be made up of both highly contextualized problems and more procedural problems. The contextualized problems will assess students’ ability to make sense of problems, reason abstractly, and construct viable arguments as well as modeling with mathematics. Procedural problems will focus on assessing students’ use of tools, attending to precision, making use of structure, and expressing regularity in repeated reasoning. The benchmark exams will be common assessments that will provide data to further drive changes in teaching strategies in order to enhance student success.

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y = f(x); \text{ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeroes when suitable factorizations are available, and showing end behavior. (+) Graph rational functions, identifying zeroes and asymptotes when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeroes, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. F-LE.4*: For exponential models, express as a logarithm the solution to } ab^x = d \text{ where } a, c, \text{ and } d \text{ are numbers and the base } b \text{ is } 2, 10, \text{ or } e; \text{ evaluate the logarithm using technology; Prove simple laws of logarithms. Use the definition of logarithms to translate between logarithms in any base. Understand and use the properties of } x = k + 2\pi n \text{ or } x = k + 360n \text{ for any integer } n. \text{ Use trigonometric equations and their solutions to model and answer questions about periodic phenomena.} \]
### Unit 8
**Description**

- Focus Standards
- Related Standards
- Learning Objectives

logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

**F-TF.5*, 6, 7**: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

**Modeling:**

*all standards marked with asterisks are modeling standards

<table>
<thead>
<tr>
<th>Unit 8 Related Standards:</th>
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<tbody>
<tr>
<td><strong>N-VM.8</strong>: (+) Add, subtract, and multiply matrices of appropriate dimensions.</td>
</tr>
<tr>
<td><strong>A-CED.2</strong>: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
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<tr>
<td><strong>A-REI.10</strong>: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
</tr>
<tr>
<td><em><em>A-SSE.1</em>, 3</em>*: Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeroes of the function it defines. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expressions for exponential functions.</td>
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### Unit 8
**Sample Investigations and Assignments**

<p>| Unit 8 |
| Sample Investigations and Assignments | Honors Extensions |</p>
<table>
<thead>
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| ● Description  
● Learning Goals  
● Key Assessments | ● Focus Standards  
● Related Standards | ● Learning Objectives | ● Sample Investigations and Assignments  
● Honors Extensions |

**S-ID.6**: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related; Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Informally assess the fit of a function by plotting and analyzing residuals; Fit a linear function for a scatter plot that suggests a linear association.
The primary goals of the EUHSD Mathematics program are as follows: first, ensure students develop a deeper understanding of mathematics; second, prepare students to take on more advanced studies in math; third, personalize the pursuit of honors and advanced coursework; fourth, to focus on the level of rigor evidenced by the work produced by students. Accordingly, students who elect to participate in and produce honors level work will receive honors credit.

Students who earn the honors designation in these courses will produce work that addresses more sophisticated questions, delve more deeply into the mathematics of the course, and satisfy the California State Standards with added rigor. The honors modification applied to some course assignments will broaden the scope or deepen the examination of the course content.

The honors option is available to all students enrolled in Math 2, Math 3S, Math 3C, and Math 4, and will take place within those classrooms. Students are provided the opportunity to opt in to the Honors requirements. Only students in Math 4 will be eligible for the UC Honors designation (pending) and receive a weighted grade. Honors students in the Math 2 and Math 3 courses will not receive a weighted grade.
I am enrolling in the HONORS OPTION for the above course. I understand that I will be expected to complete different and more challenging work than those students who do not register for the Honors Option.

I also understand that if I change my mind and decide not to complete the HONORS OPTION, I must make that decision and petition to be released from the HONORS OPTION by the (INSERT DATE or 8 weeks in) of the semester in which this contract applies.

HONORS OPTIONS drop dates are _________ and ______________ for the Fall and Spring semesters of the 2018-2019 school year, which are after the first unit exam of each semester.

A Petition to be Released from the Honors Option form requires consultation with and signatures of 1) the course teacher, 2) a parent or guardian, and 3) the student. Form must be submitted to student’s counselor.

I understand that if I decide not to continue with the HONORS OPTION but drop it by the deadline, my grade will be noted on my transcript as a regular letter grade with no penalty. However, if I do not formally drop the HONORS OPTION by the required deadline, I will remain in the Honors section and all HONORS OPTION requirements (complete or incomplete) will be counted in my final grade.

I realize that the HONORS OPTION requirements will include attendance outside of school hours for a Demonstration of Learning one time per semester.

Student Signature_________________________________________  Date________________

Teacher Signature_________________________________________  Date________________

To be completed by the Parent(s)/Guardian(s) of the student

I understand the terms of the above contract.

Parent(s)/Guardian(s) Signature___________________________________ Date________________

This form must be submitted to the student’s counselor by the 20th day of the semester. For 2018 – 2019, those dates are:

Fall Semester – September XX, 2018             Spring Semester – February XX, 2019
Escondido Union High School District

Petition for Release from Honors Option Contract

Student Name___________________________________________   ID Number ___________________________
Course Name_____________________________________________     email _______________________________
Teacher Name_____________________________________________   period___________
Semester: (circle)                     Fall     Spring                      School Year __________________

I request that I be released from the HONORS OPTION Contract in the above course for the following reason(s):
__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________

Student Signature_________________________________          Date_____________________  

**************************************************************************************************

Teacher Name_____________________________________________

The student and I have discussed this matter and I DO / I DO NOT (circle one)   agree with this decision

Teacher Signature_________________________________________  Date________________

*****************************************************************************************

To be completed by the Parent(s)/Guardian(s) of the student

I / We   DO / DO NOT  (circle one) agree with _______________________________'s decision to be released from the HONORS OPTION in this course.

Parent(s)/Guardian(s) Signature___________________________________ Date________________

This form must be submitted to the student’s counselor before the end of the first 6 week grading period (or whatever we propose)

For 2018 – 2019, those dates are:

Fall Semester – September XX, 2018  Spring Semester – February XX, 2019